Regular Polygons

Regular Polygons: Definition

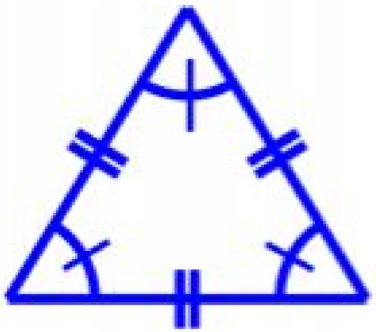
A polygon is regular if all of it’s sides are congruent and all of it’s interior angles are congruent.

Remember, congruent means identical in measurements.

So, what do we need?

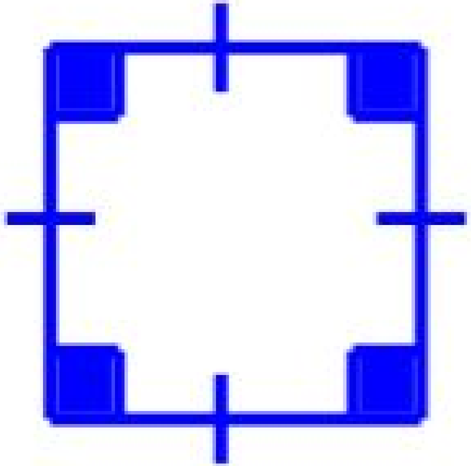
1 - Equal sides
2 - Equal angles

Equilateral Triangle



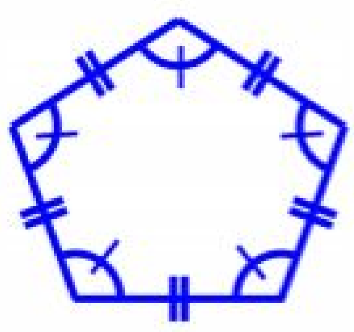
3 congruent sides
3 congruent angles

Square



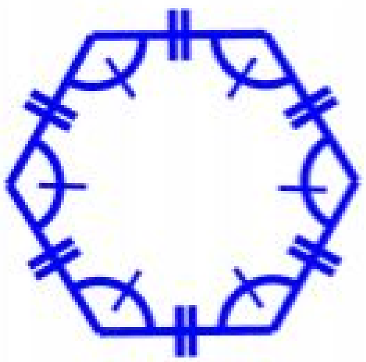
4 congruent sides
4 congruent angles

Regular Pentagon



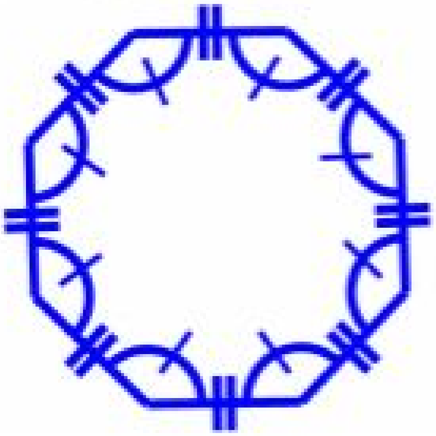
5 congruent sides
5 congruent angles

Regular HExagon



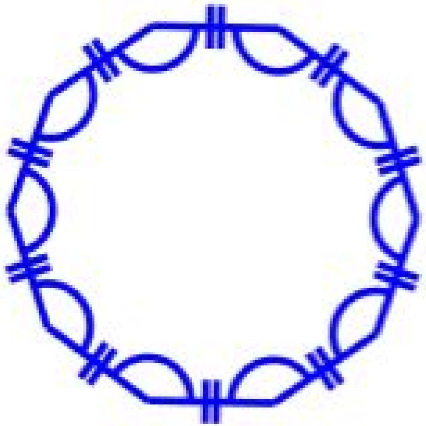
6 congruent sides
6 congruent angles

Regular Octagon



8 congruent sides
8 congruent angles

Regular Decagon



10 congruent sides
10 congruent angles

Triangles

Recall:

All the angles of a triangle, add up to a total of 180°

Polygons - Interior Angles

The sum of the interior angles of a Regular Polygon vary from Polygon to Polygon, however there is a general rule that we can use:

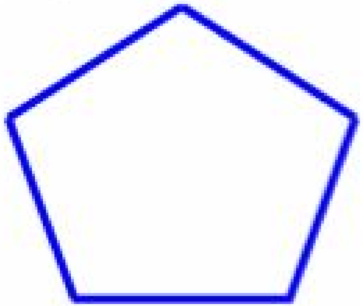
$$S = (n - 2)(180°)$$

where
S = sum of the interior angles

n = the number of sides

Example: Pentagon (n = 5)

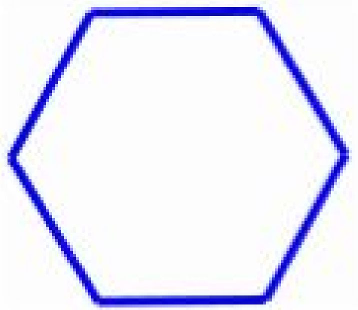
Pentagon : $n = 5$



$$S = (n - 2)(180°)$$
$$S = (5 - 2)(180°)$$
$$S = (3)(180°)$$
$$S = 540°$$

Hexagon (n = 6)

Hexagon : **n = 6**



$$S = (n-2)(180°)$$
$$S = (6-2)(180°)$$
$$S = (4)(180°)$$
$$S = 720°$$

Regular Polygons: Interior Angles

We can also determine the measure of each interior angle by dividing the sum S by the number of sides n:

$$\angle A = \frac{S}{n} \quad \text{or} \quad \angle A = \frac{(n-2)(180^\circ)}{n}$$

where

∠A = the measure of each interior angle in a regular polygon

S = is the sum of all angles

n = the number of sides

Example: Equilateral Triangles

Equilateral Triangle : **n = 3**

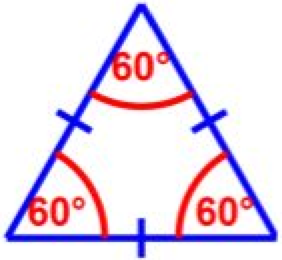
$$\angle A = \frac{(n-2)(180^\circ)}{n}$$

$$\angle A = \frac{(3-2)(180^\circ)}{3}$$

$$\angle A = \frac{(1)(180^\circ)}{3}$$

$$\angle A = \frac{180^\circ}{3}$$

$$\angle A = 60^\circ$$



Example: Square

Square: **n = 4**

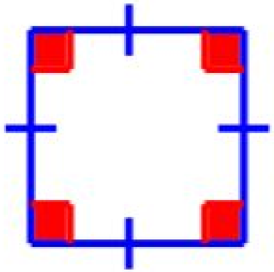
$$\angle A = \frac{(n-2)(180°)}{n}$$

$$\angle A = \frac{(4-2)(180°)}{4}$$

$$\angle A = \frac{(2)(180°)}{4}$$

$$\angle A = \frac{360°}{4}$$

$$\angle A = 90°$$



Summary

Let's summarize the results in a table.

Regular Polygon	Number of Sides	Sum of the Interior Angles	Measure of one Interior Angle
Equilateral Triangle	3	$180°$	$60°$
Square	4	$360°$	$90°$
Regular Pentagon	5	$540°$	$108°$
Regular Hexagon	6	$720°$	$120°$
Regular Octagon	8	$1080°$	$135°$
Regular Decagon	10	$1440°$	$144°$
All Regular Polygons	n	$S = (n-2)(180°)$	$\angle A = \frac{(n-2)(180°)}{n}$

Homework:

Math 3000 : pages 151 - 152 #1, 2, 3, 4(abc)

Pages 153 #5 - 10