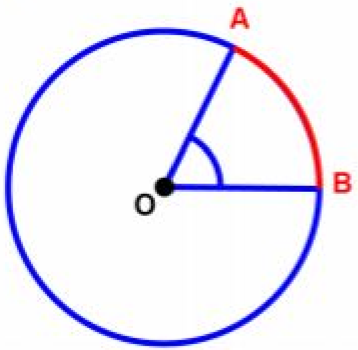
Arc Length of a Circle

The **arc length** of a circle is **proportional** to the degree measure of the **central angle** intercepting this arc.



Arc length AB $\left(\overset{\frown}{AB}\right)$ is red.

The circumference is blue.

Point O is the center of the circle in black.

The central angle for $\left(\overset{\frown}{AB}\right)$ is $\angle AOB$

There are 360º in a full circle.

The circumference (the perimeter of the circle) is the distance around a full circle.

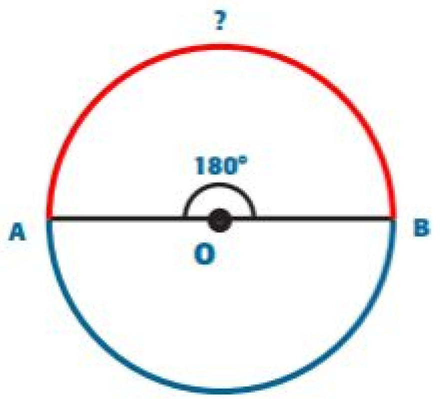
The length of $\overset{\frown}{AB}$ $\left(m\overset{\frown}{AB}\right)$ is proportional to the central angle $(m\angle AOB)$ as the circumference is proportional to 360°

As a formula,

$$\frac{m\overset{\frown}{AB}}{C} = \frac{m\angle AOB}{360°}$$

Example 1

If the circumference of the following circle is 25.12cm, calculate $m\widehat{AB}$.



First, we state the information given in the question.

$$C = 25.12cm$$

$$m\angle AOB = 180°$$

Example 1 - Continued

$$\frac{m\overset{\frown}{AB}}{C} = \frac{m\angle AOB}{360^\circ}$$

$$\frac{m\overset{\frown}{AB}}{25.12cm} = \frac{180^\circ}{360^\circ}$$

$$\frac{m\overset{\frown}{AB}}{25.12cm} = \frac{180^\circ}{360^\circ}$$

$$(360)\left(m\overset{\frown}{AB}\right) = (180)(25.12cm)$$

$$(360)\left(m\overset{\frown}{AB}\right) = (180)(25.12cm)$$

$$m\overset{\frown}{AB} = \frac{(180)(25.12cm)}{360}$$

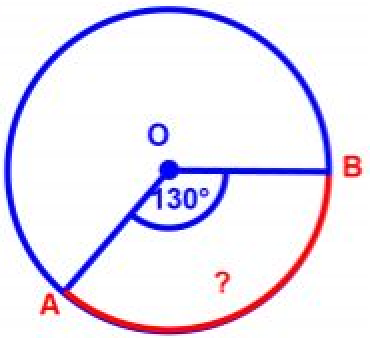
$$m\overset{\frown}{AB} = \frac{4521.6cm}{360}$$

$$m\overset{\frown}{AB} = 12.56cm$$

Note that $m\overset{\frown}{AB} = 12.56cm$ is half of the circumference $C = 25.12cm$

Example 2

If the circumference of the circle below is 43.96cm, calculate $m\overset{\frown}{AB}$ (shown in red).



$$\frac{m\overset{\frown}{AB}}{C} = \frac{m\angle AOB}{360°}$$

$$\frac{m\overset{\frown}{AB}}{43.96cm} = \frac{130°}{360°}$$

$$\frac{m\overset{\frown}{AB}}{43.96cm} = \frac{130°}{360°}$$

$$\left(m\overset{\frown}{AB}\right)(360) = (130)(43.96cm)$$

$$\left(m\overset{\frown}{AB}\right)(360) = (130)(43.96cm)$$

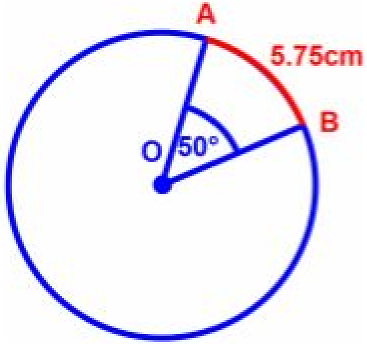
$$\left(m\overset{\frown}{AB}\right) = \frac{(130)(43.96cm)}{360}$$

$$\left(m\overset{\frown}{AB}\right) = \frac{574.8cm}{360}$$

$$m\overset{\frown}{AB} = 15.87cm$$

Example 3

If $m\widehat{AB} = 5.75cm$ in the circle below, what is the circumference of this circle?



$m\widehat{AB} = 5.75cm \quad m\angle AOB = 50°$

Example 3 - Continued

$$\frac{m\overset{\frown}{AB}}{C} = \frac{m\angle AOB}{360^\circ}$$

$$\frac{5.75cm}{C} = \frac{50^\circ}{360^\circ}$$

$$\frac{5.75cm}{C} \times \frac{50^\circ}{360^\circ}$$

$$(50)(C) = (360)(5.75cm)$$

$$C = \frac{(360)(5.75cm)}{50}$$

$$C = \frac{2070cm}{50}$$

$$C = 41.4cm$$

Homework:

MATH 3000 PAGE 173 #5, 7

PAGE 174 #8, 9

ASSIGNMENT ON MHS